SEAL

Test Booklet Number	Test - 1104	Roll Number
10799	**MATHEMATICS**	

[Time : 1 Hour] [Maximum Marks : 200]

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before you answer the questions given in this Test Booklet :

1. Answers to questions in this Test Booklet are to be given on a computerised **Answer Sheet** provided to the candidate **separately.**
2. Candidate must fill up Name, Category, Test Booklet Number, Subject Code and Roll Number in the answer sheet carefully as per instructions given.
3. This Test Booklet consists of 50 questions. All questions are compulsory and carry equal marks.
4. Each question in this Test Booklet has four possible alternative answers namely, (a), (b), (c) and (d), one of which is correct. Candidate should choose the correct answer against each question out of four alternative answers.
5. Candidate is instructed to answer the questions by **darkening (●)** with **HB Pencil** only to the circle bearing the correct answer.
6. After attempting a question, if candidate wants to change his/her answer, erase completely to change the response and re-dark another circle.
7. Marking of answer other than darkening shall be cancelled and darkening should remain within the circle or otherwise computer shall not accept during evaluation of answer-script.
8. Rough work must not be done on the Answer Sheet. Use the blank space given in the Test Booklet for rough work.
9. Candidate is to hand over the Answer sheet to the Invigilator before leaving the Examination Hall.
10. **NEGATIVE MARKING** : Each question carries 4 (four) marks for correct response. For each incorrect response, 1 (one) mark will be deducted from the total score. More than one answer indicated against a question will be deemed as incorrect response and will be negatively marked.

P.T.O.

SEAL

SEAL

MATHEMATICS

1. Let $x = \{1, 2, 3\}$, $y = \{2, 4, 6, 8\}$. Which of the following relations is a function from x to y

   (A) $f = \{(3, 6), (2, 4), (1, 2)\}$

   (B) $g = \{(1, 2), (2, 4), (2, 8), (3, 6)\}$

   (C) $h = \{(x, y) : y = 2x, x = 1,2\}$

   (D) $\Phi = \{(1, 2), (2, 2), (2, 4), (2, 6)\}$

2. The locus of the points z satisfying the condition $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{3}$ is a

   (A) parabola

   (B) circle

   (C) pair of straightlines

   (D) ellipse

3. The least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n$ is real, is

   (A) 2

   (B) 4

   (C) 8

   (D) 10

4. The determinant

$$\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & o \end{vmatrix} = 0 \text{ if}$$

   (A) a, b, c are in A.P.

   (B) a, b, c are in G.P.

   (C) a, b, c are in H.P.

   (D) $a = b + c$

5. If A is an invertible matrix, then $\det(A^{-1})$ is equal to

   (A) $\det(A)$

   (B) $\frac{1}{\det(A)}$

   (C) 1

   (D) $\frac{1}{2}$

6. The roots of the equation $x(x+2)(x^2-1) = -1$ are

   (A) equal

   (B) imaginary

   (C) real but not equal

   (D) positive

7. If $a+b+c=0$, then the equadratic equation $4ax^2+3bx+2c=0$ has

(A) roots of opposite signs

(B) imaginary roots

(C) real roots

(D) complex roots

8. A polygon has 44 diagonals, then the number of sides it has is

(A) 11

(B) 7

(C) 8

(D) 9

9. The middle term in the expansion of

$\left(1-\frac{a}{2}\right)^{14}$ is

(A) $-\frac{429}{16}a^7$

(B) $\frac{101}{192}a^6$

(C) $\frac{429}{16}a^7$

(D) $-\frac{101}{192}a^6$

10. If the sum to n terms of a series is of the form an^2+bn where a and b are costants, then the terms of the series form

(A) A. P.

(B) G. P.

(C) H. P.

(D) Not in any sequence

11. The sum of n terms of the series

$1^2+3^2+5^2+\ldots$ to n terms is

(A) $\frac{n(n+1)(2n+1)}{6}$

(B) $\left[\frac{n(n+1)}{2}\right]^2$

(C) $(2n-1)^2$

(D) $\frac{n}{3}(4n^2-1)$

12. If a, b, c are in A. P.; a, x, b are in G.P. and b, y, c are in G.P. then x^2, b^2, y^2 are

(A) H.P

(B) A.P

(C) G.P

(D) Not in any sequence

13. If $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$, then x equals

(A) -1

(B) 1

(C) 0

(D) 2

14. The value of $\sin^2 75°-\sin^2 15°$ is

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) 1

(D) 0

15. If $\alpha = 112°-30'$, then the value of $\sin\alpha$ is

(A) $\frac{\sqrt{2+2\sqrt{2}}}{2}$

(B) $=\frac{\sqrt{2+\sqrt{2}}}{2}$

(C) $\sqrt{\frac{2-\sqrt{2}}{2}}$

(D) $-\sqrt{\frac{2+2\sqrt{2}}{2}}$

16. The largest integer K for which (24) ! is divisible by 3^K is

(A) 24

(B) 15

(C) 10

(D) 8

17. The two consecutive terms in the expansion of $(3+2x)^{74}$ which have equal coefficients, are

(A) 7th and 8th

(B) 11th and 12th

(C) 30th and 31st

(D) 31st and 32nd

18. If $f(x)=\begin{cases}\frac{\sin[x]}{[x]}, & [x]\neq 0\\ 0, & [x]=0\end{cases}$

where [x] denotes the greatest integer $\leq x$, then $\lim_{x\to 0} f(x)$ is equal to

(A) 1

(B) 0

(C) -1

(D) limit does not exist

19. If p is a statement, then $\sim [p \vee (\sim p)]$ is a

(A) a tautology

(B) the dual of p. p

(C) the dual of $p'(p')'$

(D) a contradiction

20. The derivative of $\sin^{-1}x$ with respect to $\cos^{-1}\sqrt{1-x^2}$ is

(A) $\frac{1}{\sqrt{1-x^2}}$

(B) 1

(C) $\frac{1}{1-x^2}$

(D) $\tan^{-1}\frac{1}{\sqrt{1-x^2}}$

21. If $x^2+y^2 = a - \frac{1}{a}$ and $x^4 + y^4 = a^2 + \frac{1}{a^2}$, then $x^3y\frac{dy}{dx}$ equals

(A) 0

(B) 1

(C) –1

(D) 2

22. The maximum value of $\sin x(1+\cos x)$ will be at

(A) $x = \frac{\pi}{2}$

(B) $x = \frac{\pi}{6}$

(C) $x = \frac{\pi}{3}$

(D) $x = \frac{\pi}{4}$

23. If the normal to the curve $y = f(x)$, at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with the positive direction of x-axis, then $f'(3)$ is equal to

(A) –1

(B) $-\frac{3}{4}$

(C) $\frac{4}{3}$

(D) 1

24. The function $y = x^3 - 3x^2 + 6x - 17$

(A) increases everywhere

(B) decreases everywhere

(C) increases for positive x and decreases for negative x

(D) increases for negative x and decreases for positive x

25. $\int \log x \, dx$ will be equal to

(A) $x \log\left(\frac{x}{e}\right) + c$

(B) $x \log x + c$

(C) $\frac{\log x}{x} + c$

(D) $x \log\left(\frac{e}{x}\right) + c$

26. $\int_0^{2\pi} |\cos x| \, dx$ is equal to

(A) 2

(B) 4

(C) 8

(D) 12

27. $\int_0^1 \frac{1-x}{1+x} dx$ is equal to

(A) $1 - 2 \log 2$

(B) $\log 4 - 1$

(C) $2 \log 2 + 1$

(D) $\sqrt{2} \log 2 - 1$

28. The smaller area enclosed by the circle $x^2 + y^2 = 9$ and the line $x = 1$ is

(A) $2\left[9 \sec^{-1} 3 - \sqrt{8}\right]$

(B) $9 \sec^{-1} 3 - \sqrt{8}$

(C) $2\left[9 \sec^{-1} 3 + \sqrt{8}\right]$

(D) $9 \sec^{-1} 3 + \sqrt{8}$

29. $y = a e^{mx} + b e^{-mx}$ satisfies which of the following differential equation

(A) $\frac{dy}{dx} + my = 0$

(B) $\frac{dy}{dx} - my = 0$

(C) $\frac{d^2y}{dx^2} - m^2y = 0$

(D) $\frac{d^2y}{dx^2} + m^2y = 0$

30. The locus of the mid-point of the portion intercepted between the axes by the line $x \cos \alpha + y \sin \alpha = p$, where p is a perimeter is

(A) $x^2 + y^2 = 4p^2$

(B) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

(C) $x^2 + y^2 = \frac{4}{p^2}$

(D) $x^2 - y^2 = 4p^2$

31. The solution of the differential equation

$\sec^2 y\,(1+x^2)\,dy + 2x \tan y\,dx = 0$, given that $y = \frac{\pi}{4}$ at $x = 1$ is

(A) $(1+x^2) \tan y = 2$

(B) $(1+x^2) \tan y = C$

(C) $y\,(1+x^2) = 2$

(D) $\tan y = C\,(1+x^2)$

32. The distance of the point (3, 5) from the line $2x + 3y - 14 = 0$, measured parallel to the line $x - 2y = 1$ is

(A) $\frac{7}{\sqrt{5}}$

(B) $\frac{1}{\sqrt{15}}$

(C) $\sqrt{5}$

(D) $\sqrt{13}$

33. If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 2ax + 2y + 1 = 0$ touch each other, then value of a is

(A) $\pm \frac{4}{3}$

(B) 0

(C) 1

(D) $\pm \frac{5}{3}$

34. The point on the parabola $y^2 = 8x$ whose distance from the focus is 8, has x coordinate as

(A) 0

(B) 2

(C) 4

(D) 6

35. If the parabola $y^2 = 4ax$ passes through the point $(1, -2)$, then the tangent at this point is

(A) $x + y - 1 = 0$

(B) $x - y - 1 = 0$

(C) $x + y + 1 = 0$

(D) $x - y + 1 = 0$

36. The equation of the circle whose diameter is the common chord of the circles

$x^2 + y^2 + 3x + 2y + 1 = 0$ and $x^2 + y^2 + 3x + 4y + 2 = 0$ is

(A) $x^2 + y^2 + 8x + 10y + 2 = 0$

(B) $x^2 + y^2 - 5x + 4y + 7 = 0$

(C) $2x^2 + 2y^2 + 6x + 2y + 1 = 0$

(D) $x^2 + y^2 - 6x - y + 2 = 0$

37. If $\vec{a}$ and $\vec{b}$ are unit vectors such that the vector $\vec{a} + 3\vec{b}$ is perpendicular to $7\vec{a} - 5\vec{b}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

38. If $\vec{a}$ and $\vec{b}$ are two vectors such that the $\vec{a}, \vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$ then

(A) $\vec{a} \parallel \vec{b}$

(B) $\vec{a} \perp \vec{b}$

(C) Either $\vec{a} = \hat{0}$ or $\vec{b}$ or $\vec{b} = \hat{0}$

(D) $\vec{a}$ and $\vec{b}$ are both non zero vectors

39. If P, Q and R are three points with respective position vectors $\hat{i} + \hat{j}$, $\hat{i} - \hat{j}$ and $a\hat{i} + b\hat{j} + c\hat{k}$, then the points P, Q and R are collinear when

(A) $a = b = c = 1$

(B) $a = b = c = 0$

(C) $a = 1, b, c \in R$

(D) $a = 1, c = 0, b \in R$

46. Two dice are rolled one - after the other. The probability that the number on the first die is smaller than that on the second die is

(A) $\frac{1}{2}$

(B) $\frac{7}{18}$

(C) $\frac{3}{4}$

(D) $\frac{5}{12}$

47. A tosses three fair coins and B tosses two fair coins. The probability that B gets more heads than A is

(A) $\frac{3}{16}$

(B) $\frac{5}{16}$

(C) $\frac{11}{16}$

(D) $\frac{13}{16}$

48. The shortest distance between the lines $\vec{r}=\left(\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}+2\hat{k}\right)$ and $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu\left(2\hat{i}+\hat{j}+2\hat{k}\right)$ is

(A) 0

(B) $\frac{\sqrt{101}}{3}$

(C) $\frac{101}{3}$

(D) $-\frac{97}{17}$

49. $\lim_{n\to\infty}\left[\frac{1}{1-n^2}+\frac{2}{1-n^2}+\frac{3}{1-n^2}+\ldots+\frac{n}{1-n^2}\right]$ is equal to

(A) 0

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

50. The area bounded by the curve $y^2=4x$ and $x=3$ is

(A) $4\sqrt{3}$ sq units

(B) $8\sqrt{3}$ sq units

(C) $16\sqrt{3}$ sq units

(D) 12 sq units