SEAL

Test Booklet Number	Test - 0804	Roll Number
8797	**MATHEMATICS**	

[Time : 1 Hour] [Maximum Marks : 100]

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before you answer the questions given in this Test Booklet.

1. Answers to questions in this Test Booklet are to be given on a computerised **Answer Sheet** provided to the candidate **separately.**
2. Candidate must fill up Name, Category, Test Booklet Number, Subject Code, and Roll Number in the answer sheet carefully as per instruction given.
3. This Test Booklet consists of 50 questions. All questions are compulsory and carry equal marks.
4. Each question in this Test Booklet has four possible alternative answers namely, (a), (b), (c), and (d), one of which is correct. Candidate should choose the correct answer against each question out of four alternative answers.
5. Candidate is instructed to answer the questions by **darkening (●)** (with HB pencil only) to the circle bearing the correct answer.
6. After attempting a question, if candidate wants to change his/her answer, erase completely to change the response and re-dark another circle.
7. Marking of answer other than darkening shall be cancelled and darkening should remain within the circle or otherwise computer shall not accept during evaluation of answer-script.
8. Rough work must not be done on the Answer Sheet. Use the blank space given in the Test Booklet for this purpose.
9. Candidate is to hand over both the Test Booklet and Answer Sheet to the Invigilator before leaving the Examination Hall.
10. **NEGATIVE MARKING** : Each question carries 2 (two) marks for correct response. For each incorrect response, $\frac{1}{2}$ (half) mark will be deducted from the total score. More than one answer indicated against a question will be deemed as incorrect response and will be negatively marked.

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MATHEMATICS

1. The function $f(x) = \cos [\log (x+\sqrt{x^2+1})]$ is :
   a) an even function
   b) an odd function
   c) a periodic function
   d) none of these

2. If $x = 1 - 2i$, then the value of x^3+5x^2-7x+9, is :
   a) $-24-4i$
   b) $-24+4i$
   c) $24-4i$
   d) $24+4i$

3. If ω is an imaginary cube root of unity, then $(1-\omega+\omega^2)^{11}$ equals $2^{11}.k$, where k is
   a) $-\omega$
   b) ω
   c) $-\omega^2$
   d) ω^2

4. If for $0 < \theta < \pi/2$

$$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}, \text{ then}$$

   a) $a = b = 1$
   b) $a = \cos 2\theta,\ b = \sin 2\theta$
   c) $a = \sin 2\theta,\ b = \cos 2\theta$
   d) $a = b = \sin 2\theta$

5. if $\alpha \neq \beta \neq \gamma$, then one of the factors of

$$\begin{vmatrix} \alpha^3+1 & \beta^3+1 & \gamma^3+1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix} \text{ is :}$$

   a) $\alpha\beta + \beta\gamma + \gamma\alpha$
   b) $\alpha\beta\gamma + 1$
   c) $\alpha\beta\gamma - 1$
   d) $\alpha + \beta + \gamma + 1$

6. If the difference of the roots of the equation $x^2-ax+b = 0$ be same as that of the roots of $x^2-bx+a = 0$, $a \neq b$, then $a + b$ equals :
   a) -4
   b) 4
   c) 2
   d) -2

7. If α,β be the roots of the equation $p(x^2+n^2) + pnx + qn^2x^2 = 0$, then the value of $p(\alpha^2+\beta^2) + p\alpha\beta + q\alpha^2\beta^2$ is :
   a) 1
   b) -1
   c) 0
   d) n

8. The number of ways in which n books may be arranged in a row on a shelf so that two particular books shall not be together, is :
   a) $(n-1)!$
   b) $(n-2)!$
   c) $(n-1)!\,(n-2)!$
   d) $(n-1)!\,(n-2)$

9. The number of positive integers greater than a million that can be formed with the digits 2, 3, 0, 3, 4, 2 and 3 are :

a) 300

b) 480

c) 420

d) 360

10. The number of terms in the expansion of $(2x + 3y - 4z)^n$ is :

a) $n + 1$

b) $n + 3$

c) $\frac{n(n+1)}{2}$

d) $\frac{(n+1)(n+2)}{2}$

11. If x is very small in magnitude compared to a, so that $\left(\frac{a}{a+x}\right)^{\frac{1}{2}} + \left(\frac{a}{a-x}\right)^{\frac{1}{2}} = 2 + k.\frac{x^2}{a^2}$ approximately, then value of k is :

a) $\frac{1}{4}$

b) $\frac{1}{2}$

c) $\frac{1}{3}$

d) $\frac{3}{4}$

12. The sum of n terms of the series 5 + 55 + 555 + ... is :

a) $\frac{5}{81}(10^{n+1} - 9n - 10)$

b) $\frac{5}{9}(10^{n+1} - 9n - 10)$

c) $\frac{5}{99}(10^{n+1} + 9n - 10)$

d) $\frac{5}{101}(10^{n+1} - 9n + 10)$

13. Certain common numbers appear in both arithmetic progressions 18, 21, 24, ... and 16, 21, 26, ... The sum of first twenty common numbers appearing in both progressions is :

a) 3270

b) 3217

c) 3237

d) 3302

14. If (p+1)th, (q+1)th and (r+1)th terms of a G.P. are a, b and c then the value of $a^{q-r}.\ b^{r-p}.\ c^{p-q}$ is :

a) 0

b) 1

c) –1

d) 2

15. The value of $4(\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5})$ is :

a) $\frac{\pi}{2}$

b) $\frac{2}{3}\pi$

c) π

d) $\frac{3}{2}\pi$

16. If $2\cos 2\alpha_i + 4\sin 2\alpha_i = 5$ $(i = 1,2)$ then :

a) $\tan\alpha_1.\tan\alpha_2 = \frac{8}{7}$

b) $\tan\alpha_1 + \tan\alpha_2 = -\frac{8}{7}$

c) $\tan\alpha_1.\tan\alpha_2 = -\frac{3}{7}$

d) $\tan\alpha_1 + \tan\alpha_2 = \frac{8}{7}$

17. If a, b, c be the sides of a triangle such that $(a+b+c)(b+c-a) = 3bc$, then the angle A of the triangle is :

a) 60^0

b) 30^0

c) 45^0

d) 90^0

18. If p and q denote :

P : Amita is beautiful

q : Renu is weak in English

then $\sim[p \wedge \sim q]$, in words, is written as :

a) Amita is not beautiful or Renu is weak in English

b) it is not true that Amita is beautiful and Renu is not weak in English

c) it is not true that Amita is not beautiful and Renu is not weak in English

d) Amita is not beautiful and Renu is not weak in English.

19. The value of

$$\lim_{x \to 4} \frac{x^2 - 16}{\sqrt{3x-4} - \sqrt{x+4}}$$ is :

a) 32

b) $16\sqrt{2}$

c) 16

d) $8\sqrt{2}$

20. If $y = \left(\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)$, then $\frac{dy}{dx}$ equals :

a) $a^2 - x^2$

b) $\sqrt{a^2 - x^2}$

c) $\frac{1}{\sqrt{a^2 - x^2}}$

d) $\frac{-ax}{\sqrt{a^2 - x^2}}$

21. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ at x = y = 1 is :

a) 0

b) 1

c) -1

d) 2

22. The point on the parabola $y = (x-3)^2$, where the tangent is parallel to the chord joining the points (3, 0), (4, 1) is :

a) $\left(\frac{7}{2}, \frac{1}{4}\right)$

b) $\left(-\frac{1}{2}, \frac{49}{4}\right)$

c) $\left(-\frac{7}{2}, \frac{1}{4}\right)$

d) $\left(-\frac{7}{2}, \frac{-1}{4}\right)$

23. N characters of information are held on a magnetic tape, in batches of x characters each; the batch processing time is $\alpha + \beta x^2$ seconds; α, β are constants. The optimum value of x for fast processing is :

a) $\frac{\alpha}{\beta}$

b) $\frac{\beta}{\alpha}$

c) $\sqrt{\frac{\alpha}{\beta}}$

d) $\sqrt{\frac{\beta}{\alpha}}$

24. The value of c in the lagrange mean value theorem for the function $f(x) = (x-2)^2$, $2 \le x \le 3$ is :

a) $\frac{1}{2}$

b) $\frac{3}{2}$

c) $\frac{5}{2}$

d) $\frac{7}{2}$

25. If

$$\int \frac{\cos x}{\cos(x-\alpha)} dx = Ax + B \log \cos(x-\alpha) + C ,$$

then A and B are respectively :

a) $\cos\alpha$ and $\sin\alpha$

b) $\cos\alpha$ and $-\sin\alpha$

c) $\sin\alpha$ and $\cos\alpha$

d) $\sin\alpha$ and $-\cos\alpha$

26. $\int_0^{\pi/2} x\, f(\cos^2 2x)\, dx$ is equal to :

a) $\int_0^{\pi/2} f(\cos^2 2x)\, dx$

b) $\frac{\pi}{2}\int_0^{\pi/2} f(\cos^2 2x)\, dx$

c) $\frac{\pi}{4}\int_0^{\pi/4} f(\cos^2 2x)\, dx$

d) $\frac{\pi}{4}\int_0^{\pi/2} f(\cos^2 2x)\, dx$

27. The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \log\left(\frac{1+x}{4(1-x)}\right) dx$ equals :

a) $-\frac{1}{2}$

b) 0

c) 1

d) $2 \log \frac{1}{2}$

28. If $x = \int_a^y \frac{dt}{\phi(t)}$, then $\frac{dy}{dx}$ equals :

a) $\phi(x)$

b) $\phi(y)$

c) $\phi(a)$

d) $\phi(t)$

29. The area enclosed between the curve $y = \cos x$, y -axis and the line $y = x - \pi/2$ is :

a) $\frac{\pi^2}{4} - 1$

b) $\frac{\pi^2}{8} - 1$

c) $\frac{\pi^2}{4} + 1$

d) $\frac{\pi^2}{8} + 1$

30. The differential equation $y\frac{dy}{dx} + x = a$, where a is a constant, represents :

a) a family of circles having centre on y-axis

b) a family of circles having centre on x-axis

c) a family of ellipses

d) none of these

31. The order of the differential equation whose general solution is given by

$y = (c_1 + c_2) \cos(x+c_3) - c_4 e^{x+c_5}$ where c_1, c_2, c_3, c_4, c_5, are arbitrary constants, is :

a) 5

b) 4

c) 3

d) 2

32. The internal and external bisectors of the right angle of the right-angled triangle formed by the lines $x = 0$, $y = 0$ and $3x + 4y - 12 = 0$ are respectively :

a) $y - x = 0$ and $y + x = 0$

b) $3x - 2y = 0$ and $2x + 3y = 0$

c) $4x - 3y = 0$ and $3x + 4y = 0$

d) $y - 3x = 0$ and $3y + x = 0$

33. The length of common chord of the circles $(x - a)^2 + (y - b)^2 = c^2$ is :

a) $\sqrt{c^2 - (a-b)^2}$

b) $\sqrt{4c^2 - 2(a-b)^2}$

c) $\sqrt{2c^2 - (a-b)^2}$

d) $\sqrt{4c^2 - (a-b)^2}$

34. The centre of the circle passing through the origin and cutting off intercepts a and b on the x-axis and y-axis respectively is :

a) $\left(\frac{a}{2}, b\right)$

b) $\left(a, \frac{b}{2}\right)$.

c) (a, b)

4) $\left(\frac{a}{2}, \frac{b}{2}\right)$

35. On the ellipse $4x^2 + 9y^2 = 1$, one of the points at which the tangents are parallel to the line $8x = 9y$ is :

a) $\left(\frac{2}{5}, \frac{1}{5}\right)$

b) $\left(-\frac{2}{5}, \frac{1}{5}\right)$

c) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$

d) $\left(-\frac{1}{5}, \frac{\sqrt{7}}{5\sqrt{3}}\right)$

36. If the foci of an ellipse are $(\pm\sqrt{5}, 0)$ and its eccentricity is $\frac{\sqrt{5}}{3}$, then the equation of the ellipse is :

a) $4x^2 + 9y^2 = 36$

b) $9x^2 + 4y^2 = 36$

c) $36x^2 + 9y^2 = 4$

d) $9x^2 + 36y^2 = 4$

37. If the slopes of the pair of lines given by $ax^2 + 2axy + cy^2 = 0$ differ by 2 then :

a) $a^2 - c^2 + ac = 0$

b) $a^2 - c^2 - ac = 0$

c) $a^2 + c^2 + ac = 0$

d) $a^2 + c^2 - ac = 0$

38. The projections of a directed line segment on the coordinate axes are 12, 4, 3. The direction cosines of the line are :

a) $\frac{12}{13}, \frac{-4}{13}, \frac{3}{13}$

b) $\frac{-12}{13}, \frac{-4}{13}, \frac{3}{13}$

c) $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

d) $\frac{1}{13}, \frac{1}{13}, \frac{1}{13}$

39. If a plane drawn through the point $(2, 1, -2)$ also passes through the intersection of the planes, $\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) = 3$ and $\vec{r}.\left(\hat{i} + 3\hat{j} - \hat{k}\right) = 5$ then its equation is :

a) $\vec{r}.\left(3\hat{i} + 2\hat{j} - \hat{k}\right) = 10$

b) $\vec{r}.\left(\hat{i} - 2\hat{j} - \hat{k}\right) = 2$

c) $\vec{r}.\left(3\hat{i} + 2\hat{j}\right) = 8$

d) $\vec{r}.\left(2\hat{j} - \hat{k}\right) = 4$

34. The centre of the circle passing through the origin and cutting off intercepts a and b on the x-axis and y-axis respectively is :

a) $\left(\frac{a}{2}, b\right)$

b) $\left(a, \frac{b}{2}\right)$

c) (a, b)

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a) $4x^2 + 9y^2 = 36$

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d) $9x^2 + 36y^2 = 4$

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c) $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

d) $\frac{1}{13}, \frac{1}{13}, \frac{1}{13}$

39. If a plane drawn through the point $(2, 1, -2)$ also passes through the intersection of the planes, $\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) = 3$ and $\vec{r}.\left(\hat{i} + 3\hat{j} - \hat{k}\right) = 5$ then its equation is :

a) $\vec{r}.\left(3\hat{i} + 2\hat{j} - \hat{k}\right) = 10$

b) $\vec{r}.\left(\hat{i} - 2\hat{j} - \hat{k}\right) = 2$

c) $\vec{r}.\left(3\hat{i} + 2\hat{j}\right) = 8$

d) $\vec{r}.\left(2\hat{j} - \hat{k}\right) = 4$

40. The value of λ for which the plane $x + y + z = \sqrt{3}\lambda$ touches the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$, is :

a) $1 + \sqrt{2}$

b) $2 \pm \sqrt{2}$

c) $3 \pm \sqrt{3}$

d) $\sqrt{3} \pm 3$

41. If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq \vec{0}$, then for some scalar λ,

a) $\vec{a} + \vec{b} = \lambda \vec{c}$

b) $\vec{a} - \vec{b} = \lambda\left(\vec{c} - \vec{b}\right)$

c) $\vec{b} - \vec{c} = \lambda \vec{a}$

d) $\vec{a} + \vec{c} = \lambda \vec{b}$

42. If $\left|\vec{a}\right| = 3$, $\left|\vec{b}\right| = 5$, $\left|\vec{c}\right| = 7$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then the angle between $\vec{a}$ and $\vec{b}$ is :

a) $\pi/3$

b) $5\pi/3$

c) $2\pi/3$

d) $\pi/6$

43. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit coplanar vectors then the scalar triple product

$\left[2\vec{a} - \vec{b}, 2\vec{c} - \vec{c}, 2\vec{c} - \vec{a}\right]$ is :

a) 0

b) 1

c) $-\sqrt{3}$

d) $\sqrt{3}$

44. P, Q and R are three points vertically below the point O such that OP = PQ = QR. If a stone falls from rest from O, then the times of travelling the distances OP, PQ and QR are in the ratio :

a) $2 : \sqrt{3} : \sqrt{2}$

b) $\sqrt{3} : \sqrt{2} : 1$

c) $1 : \sqrt{2} - 1 : \sqrt{3} - \sqrt{2}$

d) $1 : \sqrt{3} - 1 : \sqrt{5} - 2$

45. A bullet fired into a target loses half of its velocity after penetrating $\frac{1}{4}$th. How much further will it penetrate?

a) $\frac{1}{4}$m

b) $\frac{1}{6}$m

c) $\frac{1}{12}$m

d) $\frac{1}{8}$m

46. Two forces P and Q have a resultant R. The resolved part of R on the direction of P is Q. If α be the angle between the forces, then :

a) $\sin\frac{\alpha}{2}=\sqrt{\frac{2Q}{P}}$

b) $\sin\frac{\alpha}{2}=\sqrt{\frac{P}{2Q}}$

c) $\sin\frac{\alpha}{2}=\sqrt{\frac{2P}{Q}}$

d) $\sin\frac{\alpha}{2}=\sqrt{\frac{Q}{2P}}$

47. If $n(\geq 3)$ persons are sitting in a row and two of them are selected at random, then the probability that they are NOT together is :

a) $\frac{2}{n}$

b) $1-\frac{2}{n}$

c) $\frac{2}{n-1}$

d) $1-\frac{2}{n-1}$

48. The variance of the number of heads in the tosses of a fair coin is three :

a) 1

b) $3/2$

c) $\frac{3}{4}$

d) $\frac{9}{4}$

49. The following table in the cumulative frequency distribution of the marks of the students in a class :

Marks (x) :	below 10	below 20	below 30	below 40
No. of Students (f) :	15	35	60	82

	below 50	below 60	below 70	below 80
	96	127	198	250

The class-interval of the marks having maximum frequency is :

a) 20 - 30

b) 50 - 60

c) 60 - 70

d) 70 - 80

50. A class consists of 20 boys and 30 girls. The average height of all the students is 145 cm. If the height of each of the 20 boys is increased by 5 cm. the average height of the class will be :

a) 147 cm

b) 148 cm

c) 149 cm

d) 150 cm